

Section 5.4

Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus (PART 1)

Evaluating Definite Integrals

If f is continuous on $[a,b]$, then the function

$$F(x) = \int_a^b f(t) dt$$

has a derivative at every point x in $[a,b]$ and

$$\frac{dF}{dx} = \frac{d}{dt} \int_a^x f(t) dt = f(x)$$

Find

$$\frac{d}{dx} \int_{-\pi}^x \cos t \, dt$$

and

$$\frac{d}{dx} \int_0^x \frac{1}{1+t^2} \, dt$$

More Examples

Find $g'(x)$

a.) $g(x) = \int_1^x (t^2 + 1)^3 dt$

b.) $g(x) = \int_x^2 \cos(t^2 + t) dt$

Find dy/dx

$$y = \int_1^{x^2} \cos(t) dt$$

$$\frac{dy}{dx} = \frac{d}{dx} \left(\int_1^{x^2} \cos(t) dt \right)$$

$$u = x^2 \quad \frac{du}{dx} = 2x$$

$$\frac{d}{dx} \int_1^u \cos(t) dt$$

$$\cos u \cdot \frac{du}{dx}$$
$$2x \cos x^2$$

The Fundamental Theorem with the Chain Rule

Find $g'(x)$

$$g(x) = \int_1^{\sqrt{x}} \frac{a^2}{a^2 + 1} da$$

Find dy/dx

a.) $y = \int_x^{.5} 3t \sin t \, dt$

b.) $y = \int_{2x}^{x^2} \frac{1}{2 + e^t} dt$

$$\frac{d}{dx} \int_x^7 \sqrt{\sin t} \, dt$$

$$\frac{d}{dx} \int_x^{e^x} \frac{t^2 + 1}{2t} dt$$

$$\frac{d}{dx} \int_4^x \sqrt{\sin t^3 - 4t} \, dt$$

$$\frac{d}{dx} \int_2^{\cos x} \sqrt{t} \, dt$$

$$\frac{d}{dx} \int_{3x}^5 t^3 + 1 \, dt =$$

$$\frac{d}{dx} \int_x^{2x^2} \sin t \, dt$$

Find $f'(x)$

$$f(x) = \int_2^x \sin t^2 dt$$

Find the derivative

$$g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}} dt$$

Find a function $y = f(x)$ with a derivative

$$\frac{dy}{dx} = \tan x$$

that satisfies the condition $f(3) = 5$

Find a function $f(x)$ that has a derivative of

$$\frac{\sin x}{\sqrt{x}}$$

and passes through the point $(7, -2)$

Find a function $f(x)$ that has a derivative

$$\sqrt{\cos x}$$

and passes through the point $(-\pi, 4)$

Find a function with a derivative of $2x^2 + 4x$ that passes through (2,9)

Evaluate $\int_{-1}^3 (x^3 + 1) \, dx$

Fundamental Theorem of Calculus (PART 2)

If f is continuous at every point of $[a,b]$, and if F is any antiderivative of f on $[a,b]$, then

$$\int_a^b f(x) \, dx = F(b) - F(a)$$

Also called the INTEGRAL EVALUATION THEOREM

SUBSTITUTE!!!

Find the area of the region between the curve $y = 4 - x^2$, $0 \leq x \leq 3$, and the x axis



How to Find the Total Area Analytically

To find the area between the graph of $y = f(x)$ and the x - axis over the interval $[a,b]$ analytically,

- 1.) Partition $[a,b]$ with the zeros of f**
- 2.) Integrate f over each subinterval**
- 3.) Add the absolute values of the integrals**

**Find the area of the region between the curve
 $y = x \cos 2x$ and the x-axis over the interval
 $-3 \leq x \leq 3$.**

Antiderivatives...

$$3x^2$$

$$2x^3$$

$$\cos x$$

$$\sec^2(2x)$$

$$e^x$$

$$\sqrt{x}$$

$$\frac{1}{x+1}$$

$$\frac{-1}{\sqrt{1-x^2}}$$

$$\frac{2}{x^3}$$

**Find the total area under the graph of $y = -x^2 + 4$
between $x = 0$ and $x = 5$**

$$\int_1^5 2x^2 + x - 2 \, dx$$

$$\int_{-\pi}^{\frac{\pi}{2}} \cos x \, dx$$

$$\int_0^4 2e^x \, dx =$$

Evaluate $\int_2^x t^2 + 4t - 3 \, dt$

$$= \left. \frac{t^3}{3} + \frac{4t^2}{2} - 3t \right|_2^x$$

$$= \left. \frac{t^3}{3} + 2t^2 - 3t \right|_2^x$$

$$= \left(\frac{x^3}{3} + 2x^2 - 3x \right) - \left(\frac{2^3}{3} + 2(2)^2 - 3(2) \right)$$

$$= \frac{x^3}{3} + 2x^2 - 3x - \frac{14}{3}$$

$$\frac{d}{dx} x^3 + 2x^2 - 3x - \frac{14}{3}$$

Evaluate

$$\int_0^{x^2} \cos t \, dt =$$

$$= \sin t \Big|_0^{x^2}$$

$$= \sin x^2 - \sin 0$$

$$= \sin x^2$$

If $\frac{d}{dx} \int_0^x f(t) dt = x \cos(\pi x)$

Find $f(4)$

$f'(x) =$

Suppose $\frac{d}{dx} \int_1^x g(t) dt = \frac{d}{dx} 2x^2 - 3x + 1$

Find $g'(x)$

$$g(x) = 4x - 3$$
$$g'(x) = 4$$

If $f(x) = \int_a^x e^{t^2} dt$

If $f(0) = 5$. Find $f(3)$

$$0 + C = 5$$
$$C = 5$$

$$f(x) = \int_0^x e^{t^2} dt + 5$$

$$f(3) = \int_0^3 e^{t^2} dt + 5 =$$

$$F(x) = \int_2^x t^2 + 4t \, dt$$

Find the equation of the tangent to $F(x)$ at $x = 5$

$$F'(x) = \frac{d}{dx} \int_2^x t^2 + 4t \, dt$$

$$F'(x) = x^2 + 4x$$

$$F'(5) = 5^2 + 4(5) = 25 + 20 = 45 = m$$

The graph of a continuous function f with domain $[0,8]$ is shown below. Let h be the function

defined by $h(x) = \int_1^x f(t) dt$

a.) Find $h(1)$

$$\int_1^1 f(t) dt = 0$$

b.) Is $h(0)$ positive or negative?

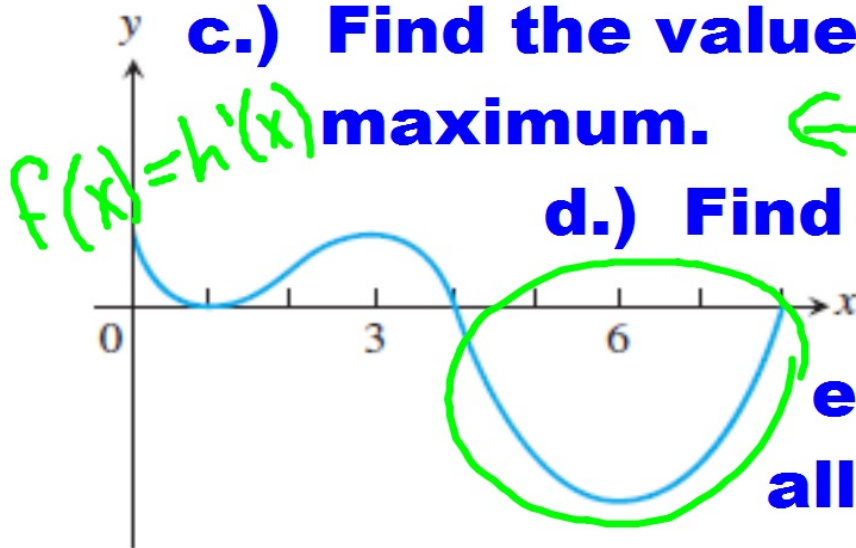
$$\int_1^0 f(t) dt = -$$

c.) Find the value of x for which $h(x)$ is a maximum.



d.) Find the value of x for which $h(x)$ is a minimum.

e.) Find the x -coordinates of all points of inflections of the graph of $y = h(x)$.



$$f(x) = \int_2^x \sin t^2 dt$$

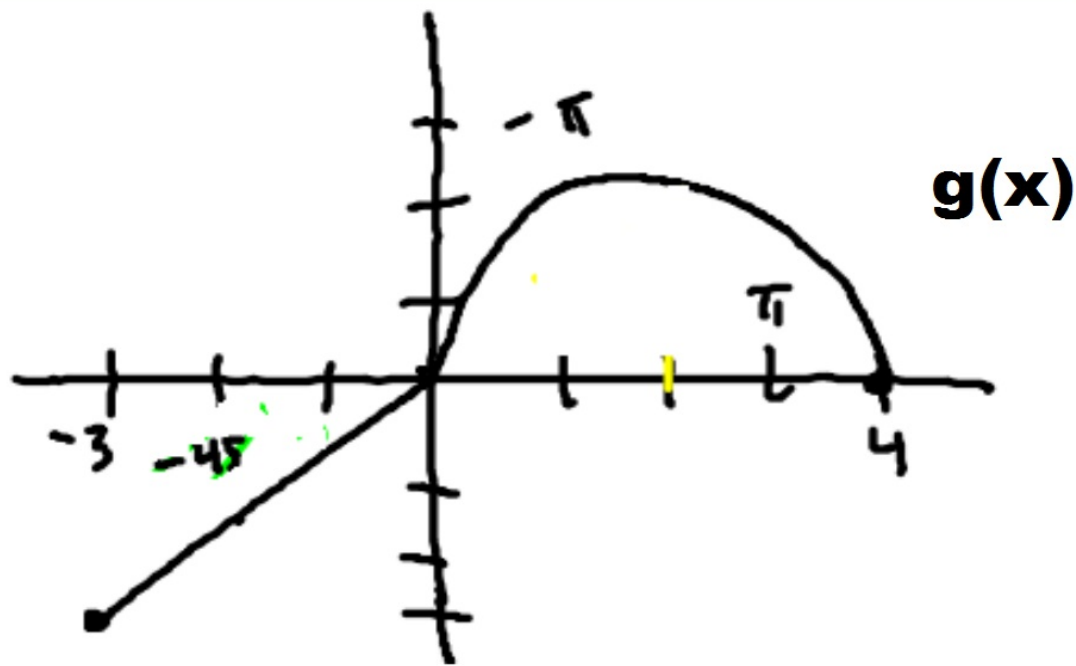
$[-\pi, \pi]$

When is $f(x)$ increasing?

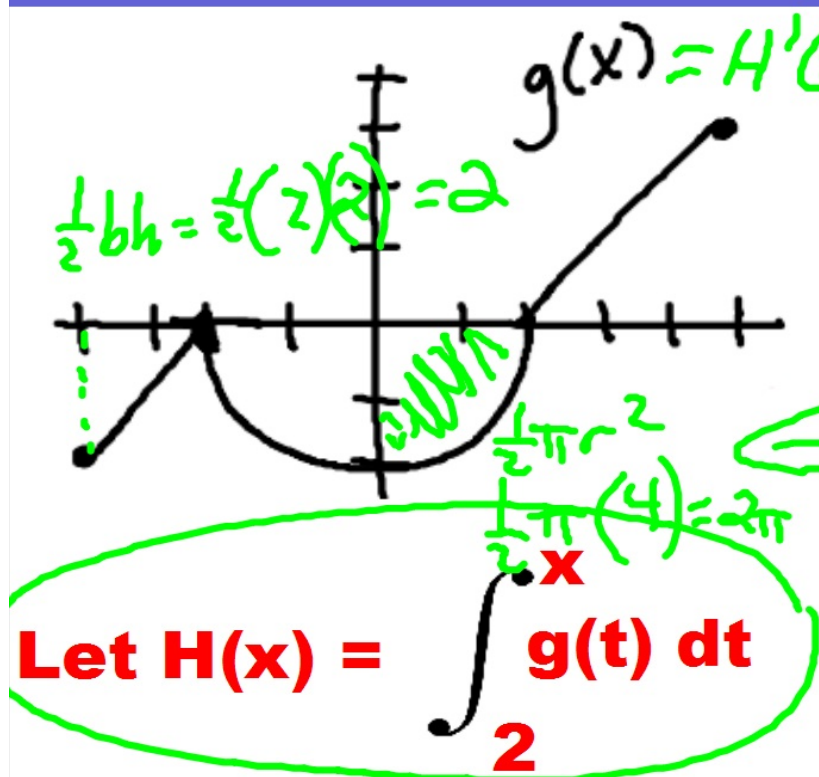
Maximum?

Is $f(x)$ concave up or concave down?

POI?



- 1.) Find $H(0) =$
- 2.) When is $H(x)$ increasing?
- 3.) When does $H(x)$ have a minimum?
- 4.) Is $x = -3$ or $x = 4$ the absolute max?
- 5.) Find $H''(-2)$
- 6.) $H'(-1) =$



1.) Find $H(2)$, $H(0)$, $H(5)$

0 π 4.5

2.) Where is $H(x)$ increasing
concave down?

(2, 5)

3.) Where does $H(x)$ have
an absolute maximum?

(-2, 0)

4.) Where does $H(x)$ have point(s) of inflection?

5.) Find $H'(0)$

6.) Find $H''(3)$

Page 303

Numbers 27 - 39

Page 304, numbers 57 and 59

Page 302, 2 - 20

Page 302, numbers 21-26





